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AVTOMATIKA I TELEMEKHANIKA

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SOME STRUCTURAL CONDITIONS THAT GOVERN THE STABILITY OF AUTOMATIC REGULATION SYSTEMS.

By M. A. Ayzerman [Eizerman]

This article determines the necessary and indispensable structural conditions under which an automatic regulation system, composed of any number of links (elements) of control operations connected in series, may be rendered stable by the proper selection of parameters.

The theorem defining these conditions is proved, and is applied to a few examples of concrete regulation systems by way of demonstration.

Vyshnegradskiy [1], in his study of direct automatic regulation in machines without the use of self-alignment (self-adjustment, self-compensation) by means of a Watt governor, formulated the following theses:

Direct automatic regulation in machines without the use of self-alignment cannot be made convergent, whatever the other parameters of the machine and the governor: 1) if there is no friction in the regulator (governor) or 2) if the regulator is astatic.

These theses revealed the influence of friction in regulators and their irregularity upon the conditions necessary for obtaining a stable process and established rational bases for the construction of direct regulation systems, thus ensuring the rapid development of regulator designs.

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Further research [2-4] showed that the first thesis was also true for direct regulation of an object having self-aligning properties, but the second thesis was not. In more complicated installations-for example, of indirect regulation-there exist systems in which the absence of friction in the regulator is sufficient to make it impossible to select parameters such that the system may be stabilized. Nevertheless, there do exist installations of indirect regulation in which the presence of friction is not a necessary condition for convergence.

The above-mentioned studies were limited to a concrete analysis of actual situations, and the systems studied were those in which the degree of the characteristic equation was not high for the most part, not higher than the sixth degree. Meanwhile, the development of methods for the structural analysis of automatic regulation systems [5-8] posed the problem of determining the limits of the necessary structural conditions of stability for a large class of systems, unhampered by any kind of limits placed on the degree of the characteristic equation. Formulated and proven below is a theorem which solves those problems applicable to systems containing any number of elements of controlled operations connected in series.

As far as the author can judge from the literature with which he is familiar, this problem has been studied in only one work [5].

However, the study in [5] is based not upon the necessary and sufficient condition that all the roots of the characteristic equation be negative, but rather upon the condition that all the coefficients of the characteristic equation be positive, which is merely necessary. Therefore, of the systems in [5] which seemed applicable to the class examined by us, the only ones to be taken up are those in which, for

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any values of the parameters, instability occurs because some of the coefficients of the characteristic equation were not positive; and, the cases not investigated are, from our viewpoint, the trivial ones where the system cannot be stabilized, despite the fact that all coefficients of the characteristic equation are positive.

1. Statement of the Problem

Let us begin by selecting two of the simplest examples related to the theme of our research.

The equations describing direct automatic regulation (Figure 1) can be reduced to the form (Note: All figures are appended to the annex):

equation of the machine: $(p+c_1)x_1 = -k_1 x_2$

equation of the governor: $(p^2+hp+c_2)x_2 = k_2 x_1$ (1)

where x_1 is the deflection (deviation) of the regulated parameter; x_2 is the displacement of the regulator clutch; $p \equiv d/dt$ is a differential operator; k_1 and k_2 are positive numbers; h, c_1, c_2 can be positive or zero. The values h, c, k_1, k_2 are the parameters of the installation.* The characteristic equation of system (1) is:

$$(p+c_1)(p^2+hp+c_2)+k_1k_2 = 0 \qquad (2)$$

The condition governing stability, by virtue of the Routh-Hurwitz criterion, is reduced to the inequality:

***Note:** The letter "h" characterizes the internal friction in the regulator c_1, the self-alignment in the regulated object (c_1 0, if the object is deprived of self-alignment); and c_2, the regulator's "static property" (c_2 0, if the regulator is astatic). The equations given by Stodola [2] or Tolle [3] amount to equation (1) with the addition of a term taking into account the machine's self-alignment and the division of each equation by the proper time constant.

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$$h^2c_1 + h(c_1^2 + c_2) - k_1k_2 > 0. \qquad (3)$$

This condition cannot be fulfilled for frictionless regulators ($h = 0$) or for astatic regulators and regulated objects non-self-aligned ($c_1 = c_2 = 0$), no matter what the values of the remaining parameters.

In the simplest indirect-regulation installation, shown in Figure 2, the regulation process is described in a linear approximation by the equations:

$$\begin{aligned} &\text{the machine:} && (p+c_1)x_1 = -k_1x_3 \\ &\text{the sensitive element:} && (p^2+hp+c_2)x_2 = k_2x_1 \qquad (4) \\ &\text{the servomotor:} && (p+c_3)x_3 = k_3x_2 \end{aligned}$$

where, besides the above notations, x_3 is the deflection of the servomotor piston and k_3 is a positive number.

The constant c_3 is positive if the servomotor has rigid feedback, shown in Figure 1, and is equal to zero if it has no feedback.

The characteristic equation of system (4) can be written thus:

$$(p+c_1)(p^2+hp+c_2)(p+c_3) + k_1k_2k_3 \qquad (5)$$

and the stability conditions are reduced to the inequalites:

$$c_1 + c_3 + h > 0$$

$$c_3(c_1h + c_2) + c_1c_2 > 0$$

$$c_1c_2c_3 + k_1k_2k_3 > 0$$

$$(c_1c_2 + c_1c_3h + c_2c_3)\left[(c_1+c_3+h)(c_1c_3+c_2+hc_1+hc_3) - (c_1c_2+c_2c_3+c_1c_2h)\right] - (c_1+c_3+h)^2(c_1c_2c_3+k_1k_2k_3) > 0.$$

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These conditions cannot be fulfilled if any one of the four cases following holds:

1) if there is no friction in the sensitive element ($h = 0$);

2) if the regulated object is deprived of self-alignment ($c_1 = 0$) and besides the sensitive element is astatic ($c_2 = 0$);

3) if the regulated object is deprived of self-alignment ($c_1 = 0$) and besides the servomotor has no feedback ($c_3 = 0$);

4) if the regulated element is self-aligning, but the sensitive element is astatic ($c_2 = 0$) and the servomotor has no feedback ($c_3 = 0$).

Both these examples are unimportant of course, and were brought forward only to illustrate our problem. They are connected with obtaining similar results applicable to more complicated systems with any order of the characteristic equation (as high as desired). Moreover, with this in view, the elements (or, as we shall call them, links) will be differentiated, not in accordance with their technical nomenclature (regulated object, sensitive elements, servomotor, etc.), but according to the form of the differential equations describing the transient processes taking place in them.

Let us limit our research to systems that satisfy the following four conditions:

1) The transient processes in any link of a system, considered aside from the remaining links, can be described by the equation

$$d(p)\,x_0 = -k x_i$$

or

$$d(p)\,x_0 = k x_i \qquad (6)$$

where d(p) is a polynomial operator with real coefficients; k is a positive number; x_0 is the generalized "output" coordinate operating on the following link; and x_i is the "input" coorinate which is the output of the preceding link.

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2) The polynomial d(p) will have the form of any of the twelve polynomials shown in Table 1.* [continue on page 7]

*Note: In preparing Table 1 and the classification of links according to type and class, the author had in view only the ease of formulating and proving the theorems to be cited later, and did not take into consideration the problem of general classification. In particular, it is possible to substitute for some of the links enumerated in Table 1 two series-connected links appearing in this table. Hence the number of "original" or "basic" links is less than twelve. For the special purposes of our work the abbreviated form of Table 1 is unsuitable. [End of note.]

Table 1

Links of Group I (static):

p^2+hp+c	(1st class)
$p+c$	(2nd ")

Links of Group II (negative staticity):

p^2+hp	(1st class)
p^2+hp-c	(1st ")
p	(2nd ")
$p-c$	(2nd ")

Links of Group III (negative staticity):

p^2	(3rd class)
p^2-c	(3rd ")
p^2-hp	(4th ")
p^2-hp-c	(4th ")

[Table 1 continued.]

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[Table 1 continued.]

Links of Group IV (static):

$p^2 + c$	(3rd class)
$p^2 - hp + c$	(4th ")

Note: The quantities h and c always represent positive constants in Table 1. [End of Table 1.]

3) The system will be a small closed circuit consisting of not less than two such links which are connected in series so that the transient processes in the coupled system can be described by the equations:*

*Note: The first equation of this system refers to the regulated object, the others to the regulator.

The directions of the readings are selected thus: x_1 —the deflection of the regulating parameter—is positive if its magnitude is greater than that on the regulated system; x_2 0 if a change in x_2 is produced by an increase in x_1; $x_3 > 0$ if a change in x_3 is produced by an increase in x_2, and so forth.

Moreover, on the right-hand side of all equations of the regulator the coefficients are positive; on the righ-hand side of the equations of the regulated object there is a minus sign, since an increase in x_n must produce a decrease in x_1. [End of note.]

$$\left.\begin{aligned} d_1(p)\,x_1 &= -k_1 x_n \\ d_i(p)\,x_i &= k_i x_{i-1} \\ i &= 2, 3, \cdots n, \end{aligned}\right\} \qquad (7)$$

and the characteristic equation will have the form:

$$\prod_{i=1}^{i=n} d_i(p) + \prod_{i=1}^{i=n} k_i = 0. \qquad (8)$$

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4) The system will contain only one link with negative staticity of the second class. There may be any number of links of other types or classes.

These conditions limit the study by excluding regulation systems with several coupled parameters and systems with action from the derivatives or having feedbacks that shunt several links at the same time, etc. With regard to more complicated systems, we shall limit ourselves to one observation in the last section of this article, but in all other sections and in the text on the theorem to be proven, for the sake of brevity in speaking about a regulation "system" or simply a "system", we shall have in mind only systems satisfying the four above-mentioned conditions, even though this may not be specifically stipulated.

We shall differentiate the links only according to the form of the polynomial d(p), naming them according to the type and class shown in Table 1. Links of the third class will also be called conservative links.

We shall divide the links enumerated in Table 1 into four groups as shown in this table.

Figures 3 and 4 give examples of the kind of links that make up the two systems studied above [equations (1) and (4)].

The purpose of this article is to determine the structural conditions necessary for stability, in other words, to select systems which can be made stable by a choice of parameters; that is, by a choice of the absolute magnitudes of the constants h, c, k in the links composing the system.

Hereafter such systems will be called structurally stable; systems without this property will be called absolutely unstable. The problems which interest us can now be formulated thus:

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What links out of the number enumerated in Table 1 can be contained in a structurally stable system?

Below, a theorem will be formulated to solve this problem, and one of the possible proofs will be developed.

2. Formulation of the Theorem

In order that an automatic regulation system satisfying the four enumerated conditions may be structurally stable, that is, made stable by the proper selection of parameters, it is necessary and sufficient that:

1) the system should not contain links of group III;

2) the system should not contain more than one link of group II and;

3) in a system containing "r" links of group IV, the degree of the characteristic equation should be higher than 4r.

3. Preliminary Observations and Summary of the Proof of the Theorem

Henceforth we shall carry on operations with the following stability criterion [6].

In the characteristic equation:

$$F(p) = \prod_{i=1}^{i=n} d_i(p) + \prod_{i=1}^{i=n} k_i \tag{9}$$

we set $p = j\omega$ (where $j^2 = -1$) and separate the real and imaginary parts

$$F(j\omega) = V(\omega) + jU(\omega)$$

then in the V, U -plane we plot the "hodograph" of the vector $F(j\omega)$.

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For the system to be stable it will be necessary and sufficient, that this hodograph,* beginning when $\omega = 0$ on the semi-axis V^+ as ω increases, should inersect the semi-axes U^+, V^-, U^-, V^+, and so forth, passing through "n" quandrants (where "n" is the degree of the characteristic equation)

*Note: Here and elsewhere the curve described by the terminus (end) of the vector during the variation of ω from zero to $+\infty$ will be called the hodograph.

in the following order: first, second, third, fourth, fifth (first), sixth (second) and so forth.

It is possible to plot a hodograph of the vector $F(j\omega)$ as follows:

1) Plot separately hodographs for the vectors $d_i(j\omega)$ of all the links;

2) Plot by a simple multiplication of the vectors $d_i(j\omega)$ the the hodograph of the vector $\prod_{i=1}^{i=n} d_i(j\omega)$.

3) Displace this hodograph parallel to the V-axis to the right of $\prod_{i=1}^{i=n} k_i$.

With such an order of operations as the basis of construction, hodographs are obtained of the vectors $d_i(j\omega)$. Figure 5 gives hodographs of the vectors $d_i(j\omega)$ of the links enumerated in Table 1.

For proof of the theorem, it is necessary to examine all conceivable variants of systems composed of any number of the links enumerated in Table 1 and to show that absolute instability always occurs when the conditions of the theorem are not fulfilled, and that in the cases stipulated in the text of the theorem the system is structurally stable. With this in view, we shall first examine systems

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composed only of links of group I and then we shall successively examine systems containing links of group I and, in addition, links of any one of groups II, III, and IV. Afterwards, we shall examine systems containing links of group I and any other links, and, at the end of the proof, systems containing any links, except links of group I. This will exhaust all possible systems that can only be composed from links appearing in Table 1.

4. Proof of the Theorem

1. A System Consisting Only of Links of Group II

The characteristic equation of such a system can be written thus:

$$D(p) + K = 0$$

where

$$D(p) = \prod_{i=1}^{i=n} d_i(p) \quad \text{and} \quad K = \prod_{i=1}^{i=n} k_i \; .$$

The modulus of "d" the vectors of links of Group I (Figure 5) is not reduced to zero for any value of ω $(0 \leqslant \omega \leqslant \infty)$. When ω increases in this interval, the argument (amplitude, or angle) of the vector increases monotonously from 0 to π in static links of the first class and to $\frac{\pi}{2}$ for static links of the second class. (see Table 1 and Figure 5). Therefore, the modulus of the vector $D(j\omega)$ likewise is not reduced to zero during the variation of ω in the interval mentioned, and its argument increases monotonously from zero to $\frac{\pi}{2}(2S_1 + S_2)$, where S_1 is the number of static links of the first class and S_2 is the number of static links of the second class. The degree "n" of the characteristic equation of the system equals $2S_1 + S_2$ and, consequently, it is always possible to assign such parameters so small a value of K that the system will satisfy the abovementioned criterion of stability.*

*Note: This statement is confirmed in practise. [7, 8].

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2. A System Consisting Only of Links of Groups I and III

The characteristic equation of such a system may take the form:

$$D_I(p)\,D_{III}(p) + K_I K_{III} = 0$$

where $D_I(p)$ is the product of all the polynomials $d_i(p)$ of all the links of group III; K_I and K_{III} are the products of the constants k_i of these same links.

The point of the characteristic $D_I(j\omega)$, corresponding to $\omega = 0$, lies on the V-axis to the right of the origin of the coordinates, and the points corresponding to a sufficiently small ω lie in the first quadrant of the V, U-plane.

If the system contains only one link of group III, then for any argument ω of the vector $D_{III}(j\omega)$ not less than π (Figure 5) the hodograph of $D_I(j\omega) D_{III}(j\omega)$, when $\omega = 0$, will intersect the V-axis downwards; and when the hodograph is displaced toward the right by any amount, it will pass at once from the first quadrant to the fourth, which is not permissible according to the stability criterion.

But if the system has more than one link of group III, the number of quadrants intersected in the required order by the hodograph of the vector $D_{III}(j\omega)$ will always be less than the degree of the polynomial $D_{III}(p)$, and the absolute instability of the system will be obvious.

Hence, in order that the system under consideration may be absolutely unstable, it is sufficient that it should contain just one link of group III.*

*Note: Some results may be obtained from this confirmation which have no direct bearing on proof of the theorem. [Note continued on p. 13]

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3. A System Consisting Only of Links of Groups I and II

The characteristic equation of such a system may be put in the form:

$$D_I(p)\,D_{II}(p) + K_I K_{II} = 0$$

where $D_I(p)$ and $D_{II}(p)$ are products of the operators of links corresponding to groups I and II, respectively, and K_I and K_{II} are the products of the constants k_i of these same links.

[Footnote on p.12 con'd.]

A negative static link of the fourth class $[d_i(p) = p^2 - hp - c]$ may be replaced by one static link of the second class $[d_i(p) = p + c_1]$ and one negative static link of the second class $[d_i(p) = p - c_2]$, if only $c_2 > c_1$. Therefore, on condition that $c_2 > c_1$, a system containing such links is unstable.

A negative static link of the third class $[d_i(p) = p^2 - c]$ may be replaced by one static link of the second class $[d_i(p) = p + c_1]$ and one negative static link of the second class $[d_i(p) = p - c_2]$, if only $c_1 = c_2$. Therefore, on condition that $c_1 = c_2$, a system containing two such links is unstable.

Of course, in both cases there occur conditions of non-absolute instability, since, by disturbing the equality $c_1 = c_2$ in the second case and the inequality $c_2 > c_1$ in the first case, it is easy to select values of c_1, c_2 and the other parameters so that the system will be stable (see below). [End of Footnote begun on p. 12.]

If, besides links of group I, a system contains only one astatic link (of the first or second class), the hodograph $D_I(j\omega)\,D_{II}(j\omega)$ passes across when $\omega = 0$ the origin of the coordinates (Figure 5).

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When the ω's are sufficiently small, the points of the hodograph will lie in the second quadrant and; with increase in ω, the argument of the vector $D_I(j\omega) D_{II}(j\omega)$ will vary monotonously. But the modulus of this vector is not reduced to zero when $\omega > 0$.

Now if we displace the hodograph $D_I(j\omega) D_{II}(j\omega)$ to the right by a sufficiently small amount $K_I K_{II}$, the displaced hodograph, starting at $\omega = 0$ on the semi-axis V^+, will later, with increasing ω, intersect the U, V — axes in the order required by the stability criterion.

In this case, the total variation of the argument of the vector $D_I(j\omega)$ during the variation of ω from zero to ∞ equals $\frac{\pi}{2}(2S_1 + S_2)$, and the total variation of the argument of the vector $D_I(j\omega) D_{II}(j\omega) + K_I K_{II}$ equals $\frac{\pi}{2}(2S_1 + S_2 + 1)$, if the system contains one astatic link of the second class and $\frac{\pi}{2}(2S_1 + S_2 + 2)$ if it contains one astatic link of the first class. The degree of the polynomial $D_I(p)D_{II}(p)$ in these cases is equal, respectively, to $2S_1 + S_2 + 1$ and $2S_1 + S_2 + 2$. Thus, in a system containing one astatic link of the first or second class in addition to links of group I, it is possible to select parameters such that it will be stable.

By analogous reasoning, it is not difficult to show that, in a system containing one negative static link (of the first or second class) in addition to links of group I, it is also possible to select parameters such that it will be stable.

But if a system, in addition to links of group I, contains any two links of group II, the agrument of the vector $D_I(j\omega)D_{II}(j\omega)$ when the ω's are sufficiently small, will be equal to or greater

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than π; that is, the curve, described by the terminus (end) of this vector, will begin for $\omega = 0$ at the origin of the coordinates or at the semi-axis V^+ to the right; then, when the ω's are small, it will run along in the third or fourth quadrant and the hodograph of the vector $D_I(j\omega)\dot{D}_{III}(j\omega) + K_I K_{II}$ will not be able to proceed since this requires the stability criterion.

If the number of links of group II is greater than 2, it is possible to select these links and their parameters such that, when the ω's are small, the argument of the vector $D_I(j\omega)D_{II}(j\omega)$ will be positive; in this event, however, the number of quadrants intersected by the hodograph of the vector $D_{II}(j\omega)$ will be less than the degree of the polynomial $D_{II}(p)$, and the system will be absolutely unstable.*

*Note: It should be noted that systems containing two or more negative static links are not included in our examination.

Hence, in a system containing one link of group II besides links of group I, it is possible to select parameters such that the system will be stable; but a system containing two or more links of group II in addition to links of group I will be absolutely unstable. ⊕ [Note on top p16]

4. A System Consisting Only of Links of Groups I and IV

Let us examine first only static links of the third class (static conservative links).

The modulus of the vector $d(j\omega)$ of a static link of the third class (Figure 5) is reduced to zero only when $\omega = \bar{\omega} = \sqrt{c}$; and the argument of the vector $d(j\omega)$, equal to zero for any $\bar{\omega} < \sqrt{c}$, becomes equal to π for ω greater than this value of $\bar{\omega}$, corresponding to the natural frequency of the link.

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Let us study first a system containing any number of links of group I and one static link of the third class. The characteristic equation of such a system can be written:

$$(p^2+c)D_I(p)+K_I k=0$$

[This note refers back to bottom of p. 15] *Note: Certain peculiar results connected with this stage of the proof of the theorem may likewise be obtained by studying the signs of the coefficients of the characteristic equation of the system (see [5]).

When $\omega=\overline{\omega}=\sqrt{c}$, the hodograph of the vector $(-\omega^2+c)D_I(j\omega)$ passes through the origin of the coordinates; and the system is unstable for any value of $K_I k$ only if, for $\omega=\overline{\omega}$, the curve intersects at the origin the V – axis in a downward direction. This can take place when the point of the curve $D_I(j\omega)$ corresponding to $\omega=\overline{\omega}$ is located in the first or second quadrant, and will not take place if this point lies in the third or fourth quadrant (Figure 6). By selecting the parameter c, it is possible to shift the point $\overline{\omega}$ to the third quadrant, if only the hodograph $D_I(j\omega)$ passes through it. Hence, by a proper selection of parameters, it is impossible to make the system stable, except in the case where the hodograph $D_I(j\omega)$ lies completely in the first or second quadrant; that is, when the polynomial $D_I(p)$ is less than the third degree and the characteristic equation is less than the fifth degree.*

*Note: Let us note the following incidental result although it has no direct bearing on our subject.

If we trace the movement (locus) of the terminus of the vector $D_I(j\omega)$ and denote by ω_0, ω_1, ω_2 .... the successive values of at which the argument of this vector is equal to $m\pi$ (where $m=1,2,3\ldots$), a system containing one link of group IV may be made stable by selecting the magnitude of Kk only on condition that the natural frequency

[Footnote continued on top of p. 17]

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[Continuation of footnote on bottom of p. 16] of the link $\bar{\omega} = \sqrt{c}$ satisfies any of the following inequalities: $\omega_1 < \bar{\omega} < \omega_2$; $\omega_3 < \bar{\omega} < \omega_4$; $\omega_5 < \bar{\omega} < \omega_6$ and so forth. If, therefore, we change the values of the natural frequency ω of this link, passing successively through all values from zero to $+\infty$, the conditions of instability and "possible stability" will alternate, and the number of these "bands" will equal the integral part of the fraction $(n_1 + 1)/2$ (if $n_1 \geqslant 2$), where n_1 is the ~~power~~ degree of the polynomial $D_I(p)$.

It is not difficult to perceive also that a system containing even two identical static links of the third class will always be unstable. [End of footnote begun at bottom of p. 16]

If the system contains "r" static links of the third class in addition to links of group I, the characteristic equation of the system may be put in the form:

$$D_I(p)D_{IV}(p) + K_I K_{IV} = 0, \tag{10}$$

where, in addition to previous notations, $D_{IV}(p) = \prod_{i=1}^{i=2} (p^2 + c_i)$, and K_{IV} is the product of all "r" constants k of the static links of the third class.

The argument of the vector $D_{IV}(j\omega)$ is always equal to zero or π, and the modulus of this vector is reduced to zero "r" times for all values of ω corresponding to the natural frequencies of static links of the third class:

$$\bar{\omega}_i = \sqrt{c_i} \quad (i = 1, 2, 3, \dots r).$$

It is impossible to make this system, consisting of any number of links of group I and r static links of the third class, stable by a proper selection of parameters, if even for one value of $\omega = \bar{\omega}$ the the hodograph $D_I(j\omega)D_{IV}(j\omega)$ passes through the origin of the coordinates, intersecting the V– axis in a downward direction.

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So that this hodograph may intersect the V – axis in an upward direction for all values of ω, it is necessary and sufficient that, for each of these values of $\bar{\omega}_i$ and for ε sufficiently small and positive, the following inequalities should be satisfied.*

$$0 < \arg \left[D_I(j\omega) D_{IV}(j\omega) \right]_{\omega=\bar{\omega}_i+\varepsilon} < \pi \qquad (11)$$
$$0 > \arg \left[D_I(j\omega) D_{IV}(j\omega) \right]_{\omega=\omega_i-\varepsilon} > -\pi .$$

*Note: In the original text of this article this part of the proof was more cumbersome. The author expresses his gratitude to Yu. I. Neymark for his suggestions.

Let us denote by φ_i the limit (when $\varepsilon \to 0$) toward which the argument $D_I(j\omega)D_{IV}(j\omega)$ approaches if $\omega = \bar{\omega}_i + \varepsilon$; and φ_{i+1} is a similar limit of the argument of the vector, if $\omega = \omega_{i+1} - \varepsilon$. If at each value of $\omega = \bar{\omega}_i$ the inequalities (9) are fulfilled, then, in order that these inequalities may also be fulfilled for $\omega = \bar{\omega}_{i+1}$, it is necessary that (Figure 7)

$$\lim_{\varepsilon \to 0} \left\{ \arg \left[D_I(j\omega) D_{IV}(j\omega) \right]_{\bar{\omega}_{i+1}-\varepsilon} - \arg \left[D_I(j\omega) D_{IV}(j\omega) \right]_{\bar{\omega}_i+\varepsilon} = \right.$$
$$= (2q+1)\pi + \varphi_{i+1} - \varphi_i ,$$

where

$$\varphi_i < \pi \quad \text{and} \quad \varphi_{i+1} < \pi \qquad (q = 0, 1, 2, \ldots).$$

The smallest value of this limit for which inequality (9) is still possible to satisfy equals $\pi + \varphi_{i+1} - \varphi_i$.

The limit (when $\varepsilon \to 0$) of the smallest variation of the argument of the vector $D_I(j\omega)D_{IV}(j\omega)$ when ω varies from zero to $\omega_r + \varepsilon$, satisfying inequality (9), is equal to:

$$\lim_{\varepsilon \to 0} \Delta = \varphi_0 + [\pi + (\varphi_2 - \varphi_1)] + [\pi + (\varphi_3 - \varphi_2)] + \cdots + [\pi + (\varphi_r - \varphi_{r-1})]$$
$$= (r-1)\pi + \varphi_0 + \varphi_r - \varphi_1$$

where φ_0 is the argument of the vector $D_I(j\omega)D_{IV}(j\omega)$ when $\omega = \bar{\omega}_1 - \varepsilon$.

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For a sufficiently small ε, $\varphi_0 \to \varphi_1 + \pi$ (Figure 8), and therefore

$$\varphi_0 + \varphi_r - \varphi_1 \longrightarrow \pi + \varphi_r$$

Thus: $$\lim_{\varepsilon \to 0} \Delta = (r-1)\pi + \pi + \varphi_r \, .$$

The value $\varphi_r > 0$ and, consequently,

$$\lim_{\varepsilon \to 0} > r\pi \, .$$

In calculating the total variation of the argument, we studied the following intervals separately: from $\omega = 0$ to $\omega = \bar{\omega}_1 - \varepsilon$; from $\omega = \bar{\omega}_1 + \varepsilon$ to $\omega = \bar{\omega}_2 - \varepsilon$, ...; from $\omega = \bar{\omega}_{r-1} + \varepsilon$ to $\omega = \bar{\omega}_r - \varepsilon$.

In each of those intervals $D_{IV}(j\omega)$ is a real number other than zero and, therefore, in each interval the variations in arguments of the vectors $D_I(j\omega)$ and $D_I(j\omega)D_{IV}(j\omega)$ are equal to each other.

Thus, it is possible to arrange the selection of the quantities $\bar{\omega}_1$, $\bar{\omega}_2$, $\bar{\omega}_3$, ..., $\bar{\omega}_r$ so that, for each value of $\bar{\omega}_i$, inequality (11) will be fulfilled, but only on condition that, during the variation in ω from zero to ∞ , the argument of the vector $D_I(j\omega)$ is changed by more than πr. For this purpose the degree of the polynomial $D_I(p)$ must be greater that 2r and the degree of the characteristic equation (10) greater than 4r.*

These conclusions are directly related to the case of a system containing static links of the fourth class. To be convinced that this is so, it is sufficient to note that we can, by arbitrarily selecting the values of the parameters in particular, make h as small as desired. When h is sufficiently small, the difference between the hodographs of the vectors d(p) $= p^2 + c$ and d(p) $= p^2 - hp + c$ has not the slightest essential effect on the behavior (course) of the hodograph of vector $D_I(j\omega)D_{IV}(j\omega)$. The difference is simply [pick up mid p. 20]

*Note: A system consisting only of links of group I is stable if $\prod_{i=1}^{i=0} k_i < k_0$, where k_0 is the so-called critical amplification factor. Static links of the third class are obtained from static

[Footnote continued on top of p. 20]

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[Continuation of footnote on p.19]

links of the first class when $h \to 0$.

The usual consideration of algebraic continuity, therefore, make the following statement possible: if, in a system consisting of links of group I only in r static links of the first class, the coefficients of h can diminish without limit then the value of k_0 will approach zero if the degree of the characteristic equation of the system is less than $(4r+1)$ and will approach a positive number if the degree of the equation is equal to or less than $(4r+1)$. It is not difficult to perceive that in this case the value of k_0 is determined by other parameters of the system and can be made sufficiently large even if "r" links of the system are conservative. [End of footnote begun bottom p.19]

that in the calculation of static links of the third class, this hodograph intersects the V - axis in an upward direction at the origin of the coordinates, while in the calculation of static links of the fourth class it intersects for small h the V - axis in an upward direction to the left of the origin of the coordinates but proportionally closer to it as h becomes smaller.

Hence, a system containing any number of links of group I and any r links of group IV cannot be made stable if the degree of the characteristic equation of the system is less than $4r+1$. Otherwise the parameters of the system may be so selected that the system will be stable.

5. Systems Containing Links of Group I and Any Other Links in Addition.

In the cases examined above, the system included links of only one of groups II, III or IV in addition to links of group I.

Now let us examine the case where the system consists of links of group I and of any number of links of other groups.

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Let us note that the problem of the possibility of obtaining a stable system was solved above by studying the behavior of the hodograph $\prod_{i=1}^{i=n} d_i(j\omega)$ near various points; in calculating links of groups II and III we were interested in the point $\omega = 0$, and in calculating links of group IV we were interested in point *

$$\omega_i = \bar{\omega}_i = \sqrt{c_i} \neq 0.$$

*Note: or the point near $\omega_i = \bar{\omega}_i$.

Moreover, it was essential to remain anywhere in the immediate vicinity of these points.

Therefore links of groups II and III, on the one hand, and links of group IV, on the other hand, predetermined as if independently of each other the possible appearance of absolute instability. Thus, any system containing links of group I among links of other groups was absolutely unstable if it contained: 1) even one link of group III, or 2) not less than two links of group II, or 3) r links of group IV when the degree of the characteristic equation of the system was less than $(4r + 1)$.

6. Systems Which Do Not Contain Links of Group I

To exhaust all possible combinations of links, we have still to examine systems consisting only of links of groups II, III, IV; that is, excluding links of group I.

We shall show that such systems are always absolutely unstable.

This conclusion is of slight importance for a system containing only links of group IV.

For systems consisting only of links of groups II and III, this conclusion follows from the same considerations employed above to prove the absolute instability of systems containing one link of group III or two links of group II besides links of group I.* The same consider-

[Note in the middle of p. 22]

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ations prove the absolute instability of a system containing any number of links of group IV and, in addition, one link of group III and/or not less than two links of group II.

Now, a system containing only one link of group II and any number of links of group IV is absolutely unstable because the hodograph of the vector $d(j\omega)$ of any link of group II has no points below the V - axis.

These considerations prove the theorem. The necessity of each one of the conditions in the text of the theorem has been separately proven above. The sufficiency of these conditions follows from the fact that, after analysis of all systems which could be constructed from the links appearing in Table I, we found that the appearance of absolute instability was possible in all cases stipulated by the theorem and we demonstrated that in all other cases it was possible to select parameters such that the system would be stable .**

[This note refers back to bottom of p. 21.]
*Note: It is assumed of course, as in all the above cases, that a system never contains less than two links.

**Note: All prior statements concerned only systems whose characteristic equation can be written as:

$$F(p) = \prod_{i=1}^{i=n} d_i(p) + \prod_{i=1}^{i=n} k_i = 0$$

where $A(p)$ is the operational polynomial and the hodograph of the vector $F(j\omega)$ is obtained by adding the vectors $\prod_{i=1}^{n} d_i(j\omega)$ and $A(j\omega)$.

A similar addition of vectors can correct the faults of the hodograph

$$\prod_{i=1}^{n} d_i(j\omega).$$

[This note refers ahead to top of p. 23.]
*Note: If it is possible without constructing the equations to tell to what group of Table I the links composing the system belong.

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5. Examples of the Application of the Theorem

In some cases, proof of the theorem makes it possible for us to obtain important technical results without employing the usual stability criteria and even without writing out the differential equations for the process. [Footnote is back on p. 22 at the bottom.]

Let us examine some of the simplest examples of this class.

In the case of direct regulation in Figure 1 examined above, the system consists of two links (Figure 3). One of them—the machine—is a link of group II if not self-aligning, but is a link of group I if self-aligning; the second—the regulator—is a link of group I only if friction and staticity are present in the regulator (first and fifth cases in Figure 3). A link of group II corresponds to an astatic regulator (third and sixth cases in Figure 3). When there is no allowance for friction in a static regulator (second and fourth cases in Figure 3), the system contains only one link of group IV.

One glance at Figure 3 is now sufficient to show all the conclusions reached at the beginning of this article by direct application of Hurwitz' criterion. They are shown in the first part of Figure 3.

Making a similar analysis of the indirectly regulated installation in Figure 2, a glance at Figure 4 is enough to form a judgment as to the possibility of satisfying the stability criteria without recourse to analyzing these criteria in each individual case.

In the third, fourth and fifth cases of Figure 4, the system contains two links of group II and, therefore, cannot be made stable.

In the second case (Figure 4), the system contains one link of group IV (i.e. $r=1$) and the characteristic equation of the system is of the fourth degree and, hence, in this case the system is always unstable.

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In the first, sixth, seventh and eighth cases of Figure 4, the system can be made stable. These results were already obtained above from application of Hurwitz' criterion.

Let us now complicate the scheme by assuming that the regulator has two-cascade amplification. Let us assign to this new servo-motor the coordinate x_4 (Figure 9) and assume that it has feedback.

According to our classification, such a servomotor is a static link of the first class (i.e. link of group I). In the first five cases of Figure 4, the structural scheme showing the regulator represented in Figure 9 may be obtained if one more link of group I (Figure 10) is included in the circuits shown in Figure 4.

In accordance with the demonstrated theorem, the system in cases 3, 4, 5 will continue to be absolutely unstable because, just as previously, it contains two links of group II. In the first case, as before, the system can be made stable by a proper selection of parameters. Previously, in the second case with one servomotor, the system was absolutely unstable. Now, however, by changing to two-cascade amplification, the stability conditions can be fulfilled in spite of the fact that there is no allowance for friction in the sensitive element, since the order of the characteristic equation has been increased by one and has become equal to five.

As a final example, let us examine the regulator shown in Figure 11 with two-cascade pneumatic amplification, approximating in design the "Ark" - type regulator. Both servomotors are of the diaphragm type. The pressure on the diaphragm of the first servomotor depends on the position of a gate valve moved by a sensitive tachometer element, and the pressure on the diaphragm of the second servomotor depends on the position of a slide valve moved by the first servomotor.

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Let us pose the two following questions:

1) Is friction in the moving parts of the regulator (the sensitive element and the servomotors) necessary to ensure stability in a system?

2) Can stability be ensured in this regulator if the static servomotors in Figure 11 are replaced by astatic servomotors?

Let us note that in this case the characteristic equation is of the ninth degree and that the solution of these two problems by analysis according to stability criteria would require cumbersome calculations.

In generalized coordinates it may be assumed that: x_1 is the variation in the angular velocity of the engine; x_2 is the shift in the clutch of the sensitive element; x_3 and x_4 are changes in pressure generated by the first and second servomotors, respectively; x_5 and x_6 are the deflections of the diaphragms of the first and second servomotors.

It is obvious from Figure 11 that the sensitive element and both servomotors in a linear idealization are conventional oscillators and correspond to static links of the first class, provided only that the sensitive element and the servomotors are static and allowance is made for friction. The regulated engine, if it is considered as self-aligning, and housings A and B* (Figure 11) correspond to static links of the second class. If we disregard the effect of the movement of the diaphragms of the servomotors upon the pressure in the housings A and B, the structural plan of the system has the form shown in Figure 12.

*Note: Pressure x_3 is set up in housing A, pressure x_5 in housing B.

The system consists only of links of groups I and, consequently, can be made stable.

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If we disregard friction in the sensitive element and both servomotors, then all three static links of the second class can be replaced by links of group IV.

In this event the number of links of group IV will be $r = 3$; hence $4r = 12$ and the degree of the equation will be $n = 9$. Therefore, friction is necessary so that the system can be made stable. Let us assume, however, that friction takes place in the servomotors only, and disregard friction in the sensitive element. Now: $r = 1$ and $4r = 4$. In this case, the system can be made stable in spite of the fact that it contains a conservative link.

Hence, to have a stable process with the regulator shown in Figure 11, friction is necessary in one of the following three elements: the sensitive element, the first or second servomotor. Let us observe that these conclusions apply also to those cases where the regulated object is not self-aligning or even has negative self-alignment (unstable)* or when the sensitive element is astatic **, if allowance is made for friction in this element alone.

[Note on top of p. 27]

If the servomotors of the governor were astitic ***, it would generally be impossible to make the system stable, since it would contain two links of group II (Figure 13, a).

An analogous case will arise if only one servomotor is astiatic and the sensitive element is also astatic (Figure 13, b) or if the

*Note: In the case in Figure 13, one of the static elements is replaced by an element of group II.

**Note: In this case an element of group II replaces one of the static elements of the first class in Figure 13. If friction in the astatic sensitive element be disregarded, the system will contain one element of group III and will be absolutely unstable, regardless of the remaining parameters of the system, including among them the friction in the servomotors.

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[This note refers back to mid p.26] ###Note: This occurs, for example, if the action of the springs in the diaphragms of the servomotors is replaced by that of weights (dotted lines in Figure 11).

regulated object is not self-aligning (Figure 13, c).

If, however, only one of the servomotors is astatic and all other links are static, the system can be made stable by the selection of proper parameters.

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[Figures follow]

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[Figures 1, 2, 5.]

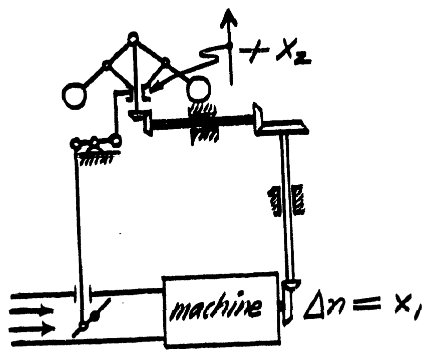

Figure 1. Schematic Diagram of Direct Regulation.

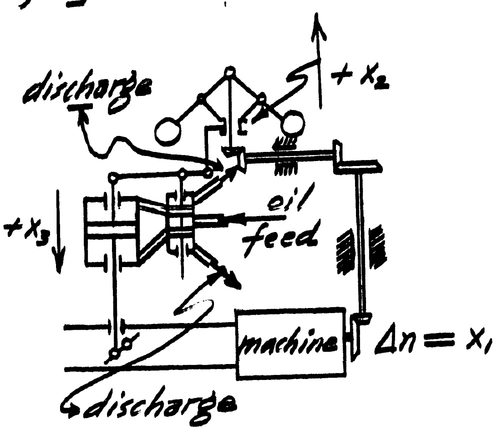

Figure 2. Indirect Regulation.

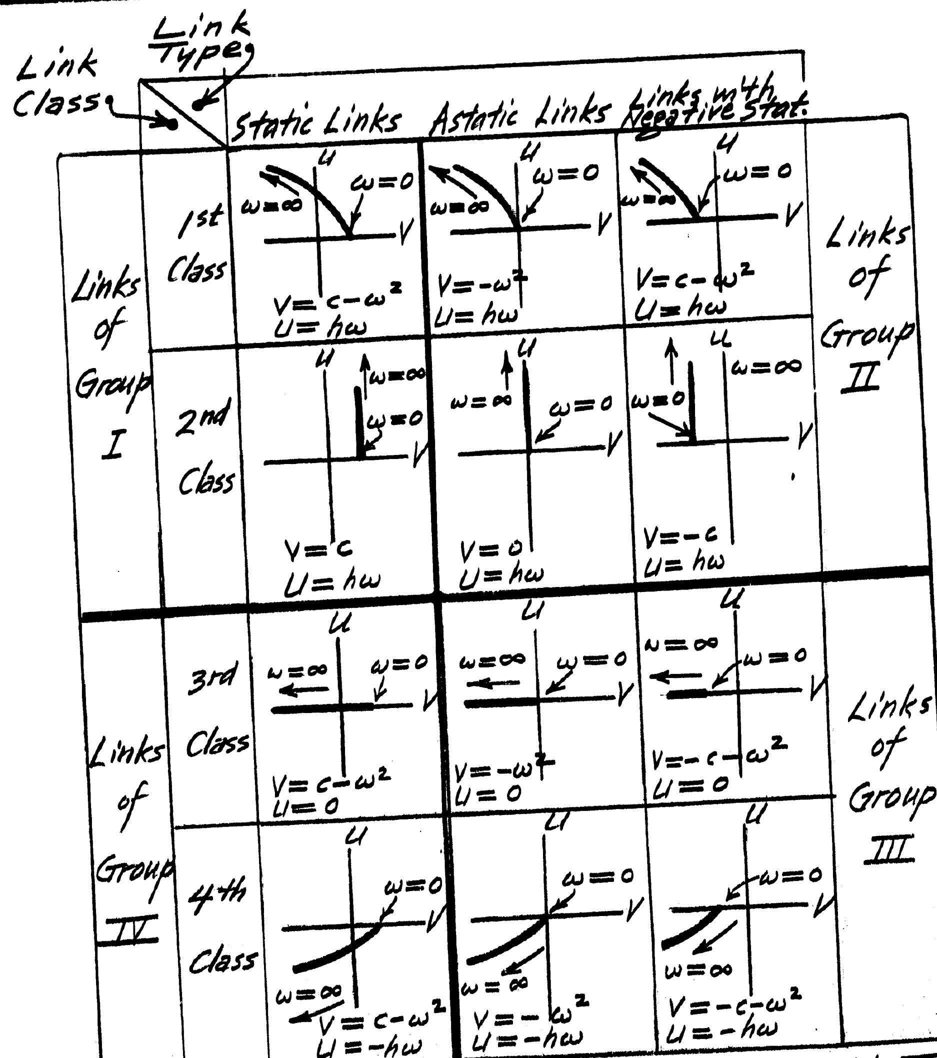

Figure 5. Hodographs of the link vectors calculated in Table 1.

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[Figures 6, 7, 8, 9, 11]

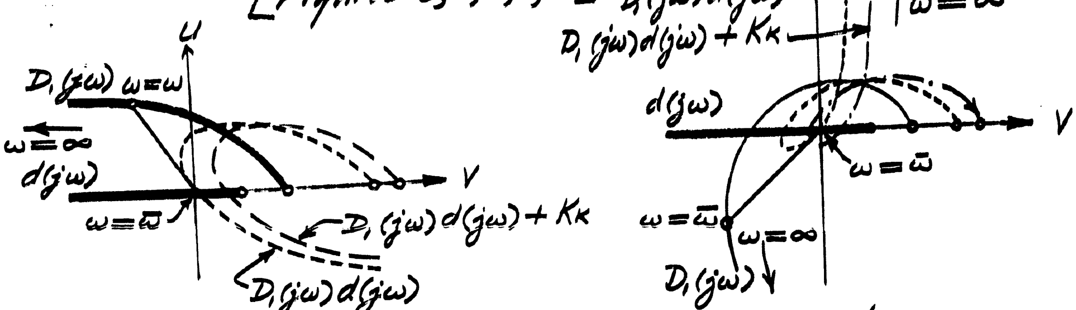

Figure 6. Construction of the hodograph of the vector $D_I(j\omega)d(j\omega)+K_k$ for a system consisting of links of Group I and one static link of the 3rd class.

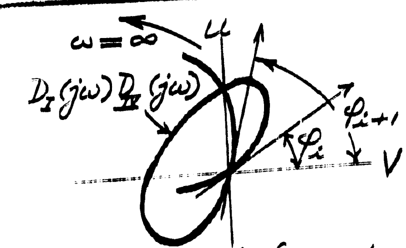

Figure 7. Graph for calculating the variation in the argument of the vector $D_I(j\omega)D_{II}(j\omega)$ for a variation of ω_i from $\bar{\omega}_i$ to $\bar{\omega}_{i+1}$.

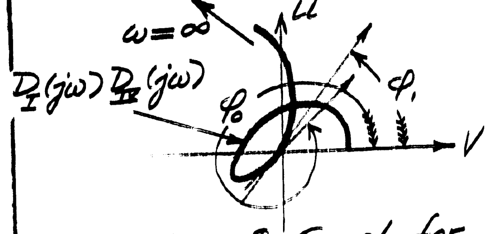

Figure 8. Graph for calculating the relation between φ_0 and φ_1.

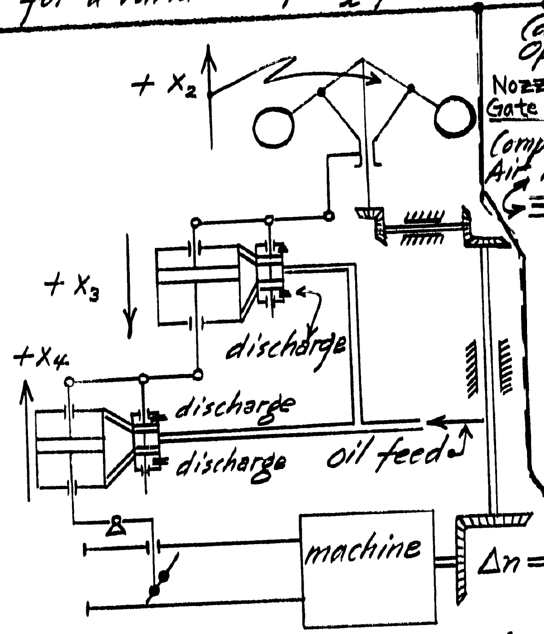

Figure 9. Schematic diagram of indirect regulation with two-cascade amplifier.

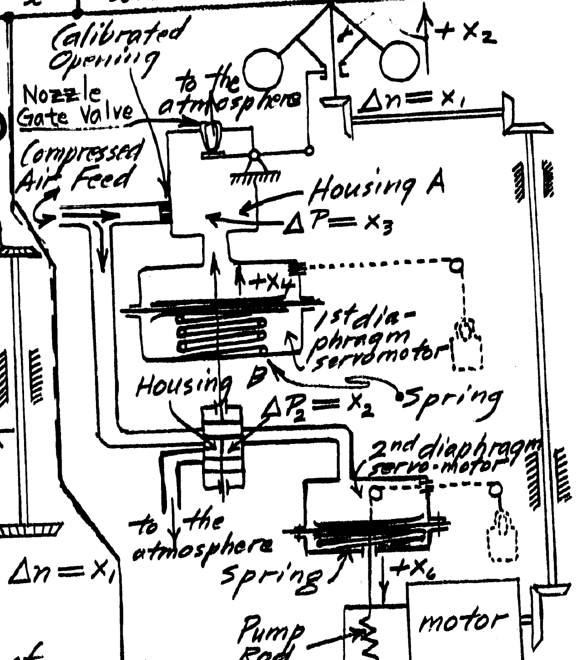

Figure 11. Schematic diagram of a regulator with two-cascade pneumatic amplifiers (servomotors — static)

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[Figure 3]

Figure 3: Structural Schemes Showing the Direct Regulation of a Machine Devoid of Self-Alignment.

1st Case: $h > 0, c_2 > 0, \quad c_1 = 0$

There is friction in the regulator.

The regulator is static.

The machine is without self-alignment.

2nd Case: $h = 0, c_2 > 0, \quad c_1 = 0$

No allowance for friction in the regulator.

The regulator is static.

The machine is without self-alignment.

3rd Case: $h > 0, c_2 = 0, \quad c_1 = 0$

There is friction in the regulator.

The regulator is astatic.

The machine is without self-alignment.

4th Case: $h = 0, c_2 > 0, \quad c_1 > 0$

There is no friction in the regulator, as in the 2nd case.

The regulator is static.

The machine has self-alignment.

5th Case: $h > 0, c_2 > 0, \quad c_1 > 0$

There is friction in the regulator.

The regulator is static.

The machine has self-alignment.

6th Case: $h > 0, c_2 = 0, \quad c_1 > 0$

There is friction in the regulator.

The regulator is astatic.

The machine has self-alignment.

[Figure 3 continued on next page.]

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[Continuation of Figure 3.]

Regulator		Regulated Machine

Cases:

1. Static Link 1st Class ⟵ Astatic Link 2nd Class
   Link of Group I — Link of Group II
   $(p^2+hp+c_2)x_2=k_2x_1$ — $px_1=-k_1x_2$
   This system can be made stable

2. Static Link 3rd Class ⟵ Astatic Link 2nd Class
   Link of Group IV — Link of Group II
   $(p^3+c_2)x_2=k_2x_1$ — $px_1=-k_1x_2$
   This system is always unstable

3. Astatic Link 1st Class ⟵ Astatic Link 2nd Class
   Link of Group II — Link of Group II
   $(p^2+hp)x_2=k_2x_1$ — $px_1=-k_1x_2$
   This system is always unstable

4. Static Link 3rd Class ⟵ Static Link 2nd Class
   Link of Group IV — Link of Group I
   $(p^2+c_2)x_2=k_2x_1$ — $(p+c_1)x_1=-k_1x_2$
   This system is always unstable

5. Static Link 1st Class ⟵ Static Link 2nd Class
   Link of Group I — Link of Group I
   $(p^2+hp+c_2)x_2=k_2x_1$ — $(p+c_1)x_1=-k_1x_2$
   This system can be made stable

6. Astatic Link 1st Class ⟵ Static Link 2nd Class
   Link of Group II — Link of Group I
   $(p^2+hp)x_2=k_2x_1$ — $(p+c_1)x_1=-k_1x_2$
   This system can be made stable

[End of Figure 3.]

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[Figure 4]

Figure 4: Structural Schemes Showing the Indirect Regulation by the Regulator Depicted in Figure 2.

1st Case: $h > 0$, $c_1 > 0$, $c_2 > 0$, $c_3 > 0$.

Machine has self-alignment

Sensitive element is static; there is friction in it

Servomotor has feedback

2nd Case: $h > 0$, $c_1 > 0$, $c_2 > 0$, $c_3 > 0$.

Machine has self-alignment

Sensitive element is static; there is no friction in it

Servomotor has feedback

3rd Case: $h > 0$, $c_1 = 0$, $c_2 = 0$, $c_3 > 0$.

Machine has no self-alignment

Sensitive element is astatic; there is friction in it

Servomotor has feedback

4th Case: $h > 0$, $c_1 = 0$, $c_2 > 0$, $c_3 = 0$.

Machine has no self-alignment

Sensitive element is static; there is friction in it

Servomotor has no feedback

5th Case: $h > 0$, $c_1 > 0$, $c_2 = 0$, $c_3 = 0$.

Machine has self-alignment

Sensitive element is astatic; there is friction in it

Servomotor has no feedback

6th Case: $h > 0$, $c_1 > 0$, $c_2 = 0$, $c_3 > 0$.

Machine has self-alignment

Sensitive element is astatic; there is friction in it

Servomotor has feedback

7th Case: $h > 0$, $c_1 = 0$, $c_2 > 0$, $c_3 > 0$.

Machine has no self-alignment

Sensitive element is static; there is friction in it

Servomotor has feedback

8th Case: $h > 0$, $c_1 > 0$, $c_2 > 0$, $c_3 = 0$.

Machine has self-alignment

Sensitive element is static; there is friction in it

Servomotor has no feedback

[Fig 4 continued]

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[Continuation of Fig 4]

Servomotor	Sensitive Element	Regulated Machine

Cases:

1. Static, 2nd, I. ← Static, 1st, I. ← Static, 2nd, I.

$(p+c_3)x_3 = k_3 x_2$ $\quad (p^2+hp+c_2)x_2 = k_2 x_1$ $\quad (p+c_1)x_1 = -k_1 x_3$

System can be made stable

2. Static, 2nd, I. ← Static, 2nd, IV. ← Static, 2nd, I.

$(p+c_3)x_3 = k_3 x_2$ $\quad (p^2+c_2)x_2 = k_3 x_1$ $\quad (p+c_1)x_1 = -k_1 x_3$

System is always unstable

3. Static, 2nd, I. ← Astatic, 1st, II. ← Astatic, 2nd, II.

$(p+c_3)x_3 = k_3 x_2$ $\quad (p^2+hp)x_2 = k_2 x_1$ $\quad p x_1 = -k_1 x_3$

System is always unstable

4. Astatic, 2nd, II. ← Static, 1st, I. ← Astatic, 2nd, II.

$p x_3 = k_3 x_2$ $\quad (p^2+hp+c_2)x_2 = k_2 x_1$ $\quad p x_1 = -k_1 x_3$

System is always unstable

5. Astatic, 2nd, II. ← Astatic, 1st, II. ← Static, 2nd, I.

$p x_3 = k_3 x_2$ $\quad (p^2+hp)x_2 = k_2 x_1$ $\quad (p+c_1)x_1 = -k_1 x_3$

System is always unstable

6. Static, 2nd, I. ← Astatic, 1st, II. ← Static, 2nd, I.

$(p+c_3)x_3 = k_3 x_2$ $\quad (p^2+hp)x_2 = k_2 x_1$ $\quad (p+c_1)x_1 = -k_1 x_3$

System can be made stable

7. Static, 2nd, I. ← Static, 1st, I. ← Astatic, 2nd, II.

$(p+c_3)x_3 = k_3 x_2$ $\quad (p^2+hp+c_2)x_2 = k_2 x_1$ $\quad p x_1 = -k_1 x_3$

System can be made stable

8. Astatic, 2nd, II. ← Static, 1st, I. ← Static, 2nd, I.

$p x_3 = k_3 x_2$ $\quad (p^2+hp+c_2)x_2 = k x_1$ $\quad (p+c_1)x_1 = -k_1 x_3$

System can be made stable

[End of Figure 4]

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[Figure 10]

Figure 10: Structural Scheme Showing Indirect Regulation With Two-Cascade Amplifiers.

1st Case: $h > 0 \quad c_1 > 0 \quad c_2 > 0 \quad c_3 > 0$

Machine has self-alignment;
sensitive element is static; has friction.
First servomotor has feedback.

2nd Case: $h = 0 \quad c_1 > 0 \quad c_2 > 0 \quad c_3 > 0$

Machine has self-alignment;
sensitive element is static, but has no friction.
First servomotor has feedback

3rd Case: $h > 0 \quad c_1 = 0 \quad c_2 = 0 \quad c_3 > 0$

Machine has no self-alignment;
sensitive element is isochronous; has friction.
First servomotor has feedback.

4th Case: $h > 0 \quad c_1 = 0 \quad c_2 > 0 \quad c_3 = 0$

Machine has no self-alignment;
sensitive element is static; has friction.
First servomotor has no feedback.

5th Case: $h > 0 \quad c_1 > 0 \quad c_2 = 0 \quad c_3 = 0.$

Machine has self-alignment;
sensitive element is isochronous; has friction.
First servomotor has no feedback.

[Figure 10 continued on p. 33.]

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[Continuation of Fig. 10.]

2nd Servo	1st Servo	Sensitive Element	Regulated Machine

(Cases:)

1. Static, 2, I. ← Static, 2, I. ← Static, 1, I. ← Static, 2, I. ←

   $(p+c_4)x_4 = k_4x_3$ $(p+c_3)x_3 = k_3x_2$ $(p^2+hp+c_2)x_2 = k_2x_1$ $(p+c_1)x_1 = -k_1x_4$

   System can be made stable

2. Static, 2, I. ← Static, 2, I. ← Static, 2, IV. ← Static, 2, I. ←

   $(p+c_4)x_4 = k_4x_3$ $(p+c_3)x_3 = k_3x_2$ $(p^2+c_2)x_2 = k_2x_1$ $(p+c_1)x_1 = -k_1x_4$

   System can be made stable

3. Static, 2, I. ← Static, 2, I. ← Astatic, 1, II. ← Astatic, 2, II. ←

   $(p+c_4)x_4 = k_4x_3$ $(p+c_3)x_3 = k_3x_2$ $(p^2+hp)x_2 = k_2x_1$ $px_1 = -k_1x_4$

   System cannot be made stable

4. Static, 2, I. ← Astatic, 2, II. ← Static, 1, I. ← Astatic, 2, II. ←

   $(p+c_4)x_4 = k_4x_4$ $px_3 = k_3x_2$ $(p^2+hp+c_2)x_2 = k_2x_1$ $px_1 = -k_1x_4$

   System cannot be made stable

5. Static, 2, I. ← Astatic, 2, II. ← Astatic, 1, II. ← Static, 1, I. ←

   $(p+c_4)x_4 = k_4x_4$ $px_3 = k_3x_2$ $(p^2+hp)x_2 = k_2x_1$ $(p+c_1)x_1 = -k_1x_4$

   System cannot be made stable

[End of Figure 10.]

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Figure 12. Structural Scheme Showing Regulation by the Regulator Depicted in Figure 11.

2nd Servo	Housing B	1st Servo	Housing A	Sensitive Element	Motor
← Static link ←	Static link ←	Static link ←	Static link ←	Static link ←	Static link ←
1st class, I.	2nd class, I.	1st class, I.	2nd class, I.	1st class, I.	2nd class, I.
$(p^2+h_6p+c_6)x_6 = k_6x_5$	$(p+c_5)x_5 = k_5x_4$	$(p^2+h_4p+c_4)x_4 = k_4x_3$	$(p+c_3)x_3 = k_3x_2$	$(p^2+h_2p+c_2)x_2 = k_2x_1$	$(p+c_1)x_1 = -k_1x_6$

Figure 13. Structural Scheme of the Regulator Depicted in Figure 11, With Astatic Links.

	2nd Servo	Housing B	1st Servo	Housing A	Sensitive Element	Motor
(a)	Astatic link ←	Static link ←	Astatic link ←	Static link ←	Static link ←	Static link ←
	1st class, II.	2nd class, I.	1st class, II.	2nd class, I.	1st class, I.	2nd class, I.
	$(p^2+h_6p)x_6 = k_6x_5$	$(p+c_5)x_5 = k_5x_4$	$(p^2+h_4p)x_4 = k_4x_3$	$(p+c_3)x_3 = k_3x_2$	$(p^2+h_2p+c_2)x_2 = k_2x_1$	$(p+c_1)x_1 = -k_1x_6$
(b)	Static link ←	Static link ←	Astatic link ←	Static link ←	Astatic link ←	Static link ←
	1st class, I.	2nd class, I.	1st class, II.	2nd class, I.	1st class, II.	2nd class, I.
	$(p^2+h_6p+c_6)x_6 = k_6x_5$	$(p+c_5)x_5 = k_5x_4$	$(p^2+h_4p)x_4 = k_4x_3$	$(p+c_3)x_3 = k_2x_2$	$(p^2+h_2p)x_2 = k_2x_1$	$(p+c_1)x_1 = -k_1x_6$
(c)	Static link ←	Static link ←	Astatic link ←	Static link ←	Static link ←	Astatic link ←
	1st class, I.	2nd class, I.	1st class II.	2nd class, I.	1st class, I.	2nd class, II.
	$(p^2+h_6p+c_6)x_6 = k_6x_5$	$(p+c_5)x_5 = k_5x_4$	$(p^2+h_4p)x_4 = k_4x_3$	$(p+c_3)x_3 = k_3x_2$	$(p^2+h_2p+c_2)x_2 = k_2x_1$	$px_1 = -k_1x_6$

Figures 12, 13

– F –

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END

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